THE MASS OF THE ELECTRON

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The mass of the electron, m_0, may be estimated by means of a single experiment. We use in this estimate two theoretical formulas (*a*) Bragg's formula for the reflection of x-rays by a crystal,

$$\lambda = 2d \sin \Theta$$

and (*b*) Bohr's formula for the Rydberg constant,

$$cR_\infty = \frac{2\pi^2 e^4 m_0}{h^3}.$$

For the last sixteen years or so I have been using for the grating constant of calcite (100 planes) $d = 3.028 \times 10^{-8}$ cm. A few years ago Birge, in his extensive paper on the general physical constants (*Physical Review*, Supplement, July, 1929), came to the conclusion that for calcite, $d = 3.0283 \times 10^{-8}$ at 20°C. and, correcting for various orders of reflection, that, for the first order, the effective grating space, $d = 3.0279 \times 10^{-8}$. Very recently, Bearden (*Physical Review*, December 15, 1931) published a new estimation of this grating constant of calcite and gave the value 3.0278 Å at 18°C. for the reflection of x-rays in the first order. The slight variation for different orders corresponds with the observations published by Patterson and the writer (*Physical Review*, 1920), following Stenstrom. I am inclined to think, therefore, that the value of d which I have been using is the best four-figure value for the first few orders of reflection.

Some years ago we measured the value of h by means of the reflection of x-rays by a calcite crystal (Duane and Blake, *Physical Review*, December, 1917, p. 624; Duane, Palmer and Yeh, *Proceedings of the National Academy of Sciences*, August, 1921, pp. 237–241). In these researches we used the above-mentioned Bragg formula with $d = 3.028 \times 10^{-8}$ cm.

The formula for calculating h is the quantum equation of Einstein, $Ve = hc/\lambda$, which applies to the short wave-length limit of the continuous x-ray spectrum (Duane and Hunt, *Physical Review*, August, 1915, p. 111), and for which the Bragg equation gives λ.

In estimating the value of "h" by means of x-ray reflections we assume a value for the electron charge, "e." Since, however, $e^{1/3}$ enters into the calculation of the grating constant of calcite, what we really determine by means of the x-ray reflection measurements is $e^{4/3}/h$.

In each of the two measurements of h above referred to the experiments consisted only in the measurement of an angle and that of a voltage. The angle was measured in the ordinary way, by means of a divided circle and vernier. The voltage was measured in one experiment by comparing it with the voltage of a standard cell by means of a potentiometer, the manganin resistances of which were very carefully measured and the insulation of the circuits was made similar to the insulation used in delicate electrostatic experiments. This experiment gave $e^{4/3}/h = 5.689 \times 10^{13}$.

The estimate of the percentage error made in measuring something usually amounts to nothing more than a guess, especially when the number of values obtained is very small. I have usually thought that the above value of $e^{4/3}/h$ may contain a probable error of approximately $1/20$ of 1%. In estimating this value of $e^{4/3}/h$ the very small correction suggested by Birge for the fact that the absolute value of the volt instead of the commercial value should be used has been made.

Using the above value of $e^{4/3}/h$ and using also Bohr's expression for the Rydberg constant we can calculate immediately the mass, m_0, of the electron. Bohr's equation for the Rydberg constant is

$$cR_\infty = \frac{2\pi^2 e^4 m_0}{h^3}.$$

The errors in the measurement of the Rydberg constant and of the velocity of light are so small that we may neglect them and we may take for the value of cR_∞

$$cR_\infty = 3.2899 \times 10^{15}.$$

Calculating m_0 from these two expressions and from our experiment on x-ray reflection we get $m_0 = 9.054 \times 10^{-28}$. The mass, m_0, estimated in this way depends only on one set of experiments, the measurements of h, assuming the Rydberg constant and Bragg's expression for λ to be given.

If we take the experimentally determined value of e/m_0 we can calculate the value of e. We may also determine h from the x-ray experiments. In 1930, Charlotte T. Perry and Professor E. L. Chaffee measured the value of e/m_0 without using the electric or the magnetic deflection of electron particles (*Physical Review*, **36**, September 1, 1930, pp. 904–918). These experiments were carried out in our Cruft Laboratory and consisted in the comparison of the velocity of electrons with the velocity of very short Hertzian waves. The electron rays were produced by the constant voltage coming from our very high-tension storage battery. The voltage applied was measured exactly as we had measured voltages in our above-mentioned research on the value of h, comparing it with a standard cell by means of a potentiometer including the same very high resistances which we had previously used. These resistances were re-checked and found to have changed by a negligible amount during the twelve years. Using the value of the electron energy as given by the voltage, and the velocity measured, the authors determined $e/m_0 = 1.7606 \times 10^7$ abs. e. s. u. Using this value of e/m_0 and the above value of m_0, e turns out to be $e = 1.5940 \times 10^{-20}$ e. m. u. $= 4.779 \times 10^{-10}$ e. s. u. This value of e differs from that given by Millikan by less than his estimate of his experimental errors. We may also re-calculate h from our experiments and find that $h = 6.568 \times 10^{-27}$. These values of e and h lie close to those recently estimated by W. N. Bond (*Philosophical Magazine*, September, 1931).

Since the date on which Professor Chaffee and Miss Perry published their researches several other measurements of e/m_0 have been made. J. S. Campbell and W. V. Houston made a measurement and they presented their results to the Physical Society at a meeting held in Los Angeles, California (*Physical Review*, **37**, January 15, 1931, p. 228). In their researches the ratio e/m_0 is determined from the Zeeman effects in two spectral lines, one of zinc and the other of cadmium. They think the measurements with the zinc line are slightly more accurate than those with the cadmium line. For the zinc line value of this ratio they give $e/m_0 = 1.7577 \times 10^7$ e. m. u. Using this value of e/m_0 and our value of the mass of the electron charge, the charge e, in e. s. u. turns out to be $e = 4.771 \times 10^{-10}$, a value slightly less than the average value given by Millikan. With this value of e we may re-calculate from our experiments the value of h, which turns out to be $h = 6.553 \times 10^{-27}$.

The most recent estimate of the value of e/m_0 made in Germany is described by Fritz Kirchner in the *Annalen der Physik* (5 volge, 1932, Band 12, Heft 4, pp. 503–508). Kirchner gives for the value of e/m_0

$$e/m_0 = 1.7585 \times 10^7.$$

This value lies between those used above in making the two calculations of e and h. Using it in the calculations we get the following values of e and of h:

$$e = 4.773 \times 10^{-10} \text{ e. s. u.}$$
$$h = 6.557 \times 10^{-27}.$$

These values appear to be very nearly correct according to our present knowledge of electrical measurements.

To sum up, the rest mass m_0 of the electron has been calculated from the experiments that we performed a number of years ago in determining the value of h. In this estimate of m_0 ($m_0 = 9.054 \times 10^{-28}$) the following three theoretical formulas have been used: (*a*) Bragg's formula for the reflection of x-rays by calcite, $d = 3.028 \times 10^{-8}$ cm.; (*b*) Bohr's formula for the Rydberg constant, the value of the Rydberg constant being $cR_\infty = 3.2899 \times 10^{15}$ cm.; and (c) Einstein's quantum equation. The experimental values of e and of h are not used in this calculation of m_0, nor are the results of any other experiments than ours on the reflection of x-rays by calcite, and the experimental determinations of the fundamental constants d and cR_∞. From our value of m_0 and from measurements by others of e/m_0 we can calculate first e and then h.

AN INSTRUMENT FOR THE PHOTOMETERING OF THE NEW X-RAY LINES

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In the PROCEEDINGS of the National Academy of Sciences for January, 1932, appears an article in which I describe experiments on the new K series x-ray lines, of which I spoke in a paper presented to the American Physical Society at its New York meeting in February, 1931. The end of the article contains several photometer curves of the β doublet, the γ line and a few of the new lines belonging to the molybdenum K series spectrum. The photometer now existing in the Jefferson Laboratory produced some of these curves, and for the other curves it gives me great pleasure to thank Dr. DuMond of the California Institute of Technology.

Recently I have designed and had set up in our new Physical Research Laboratory a photometer for drawing curves of x-ray spectra. The photometer contains some new features. Figure 1 represents the instrument. At the extreme right of the figure appears a box of considerable size, which contains a high powered electric lamp (a one-kilowatt lamp, for instance). The photographic negative of the x-ray line spectrum to be examined by the photometer lies just below "*a*." An accurately ground lens system made by Leits for producing microphotographs lies at "*b*" and projects the spectrum lines to the vertical front side of a small metal box at "*c*." In this particular case the two lines of the β doublet in the K series of molybdenum appear at "*c*." The photograph resembles one of those shown in the previous article (New Lines in the K Series of X-rays, The PROCEEDINGS of the National Academy of Sciences, Vol. 18, No. 1, pp. 63–68, January, 1932). The two β lines appear clearly separated from each other although the difference between their wave-lengths amounts to only 0.000563 Å.